
Harvesting the Wind: The Physics of Wind Turbines

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Integrative Exercise
April 13, 2005
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> "The pessimist complains about the wind; the optimist expects it
> to change; the realist adjusts the sails." --William Arthur Ward

Abstract

Alternative energy sources have become much more necessary as fossil fuels are depleted and pollute the environment. Wind energy is one of the most cost effective of all types of renewable energy. It does not create pollution or waste and the fuel, wind, is not used faster than it is produced. However, to make wind a viable source of energy—electricity in particular—careful design of wind-capturing machines is necessary. A variety of principles of physics are used to create wind turbines that can efficiently capture energy from the wind. This paper discusses the wind and how the parts of a wind turbine—blades, rotor, gears, generator, and electronics—operate to capture wind energy and turn it into electricity. Focus is given to horizontal axis wind turbines (HAWT), the most common and efficient type of wind energy conversion device.

Introduction

Recent concerns about the environment and the need for cleaner, renewable energy resources have brought about several innovative exploitations of the earth's energy supplies. Most of the energy available on earth, with the exception of geothermal, tidal, and nuclear, comes from the sun, some more directly than others. The use of solar paneling for heat is one of the most direct uses, followed by photovoltaic cells, wind, biomass, and fossil fuels. Currently, wind energy is one of the least expensive of the alternative/renewable energy sources and is becoming more affordable as the technology improves and infrastructure develops.[1] The goal is to "find the right combination of size, shape, materials, and location that will produce the most electricity for the least cost."[2] The employment of efficient wind turbines, which convert the mechanical energy of the wind into usable electrical energy, requires extensive use of physics. The following will examine physics principles exercised in the creation and use of wind turbines.

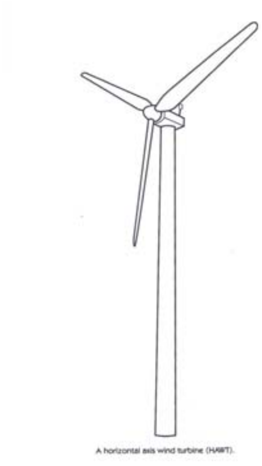

Figure 1 **A typical wind turbine. (Carless, 1993)**

A wind turbine is essentially a very large, inverse fan: the wind produces electricity instead of electricity producing wind. However, because wind turbines run 'backwards' and are several thousand times larger than most fans (~85-400 tons), they are much more complicated, especially since it is necessary to get the greatest efficiency and quality at the least cost. Modern wind turbines range from about 40 – 80m in height, 50 – 85m in span, and 850 kW to 4.5MW in power. They usually have three blades and almost always have a horizontal axis shaft (like old European windmills) as shown on the left in Figure 2, but there are those with a vertical axis, as

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[1] Jennifer Carless, *Renewable Energy*. (Walker and Company, New York, 1993), p. 43.

shown on the right in Figure 2. For a horizontal axis wind turbine (HAWT), the plane of the rotor (i.e. the blades and the hub) turns so that the wind is perpendicular to it and can flow around the blades to make them rotate around the hub. The horizontal shaft of the rotor also turns and runs a generator to which it is connected through a gearing system. The generator creates electricity from the mechanical energy and sends the resulting current down the tower to the electrical grid, which supplies electricity to the consumers. Each of these components will be explained in more detail, starting with where the powering wind originates. I will then discuss how the power in the wind transfers to the turbine, in particular focusing on the aerodynamic aspects of energy collection. I will briefly cover loads and fatigue issues, followed by the gear system, the generator, and the other electronics that lead to the delivery of high quality electricity.

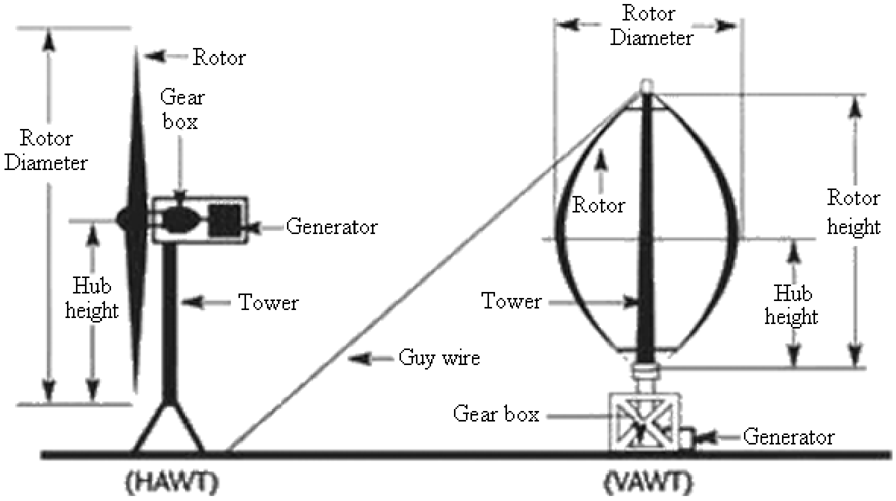

Figure 2 **The two main types of turbines, horizontal axis (HAWT) on the left and vertical axis on the right (VAWT) with their respective components labeled. The axis refers to the orientation of the rotor shaft. HAWTs are more common because they are intrinsically more efficient. (http://www.canren.gc.ca/tech_appl/index.asp?CaID=6&PgID=219)**

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[2] James L. Schefter, *Capturing Energy from the Wind.* (NASA Scientific and Technical Info Branch, Washington D.C., 1982), p. 18

The Wind

Wind energy comprises only a small amount of the total energy that reaches the earth. About 1.74 x 10^{17} watts of power from the sun contact the earth. Each year this is 160 times the total energy in the world's reserves of fossil fuels. Only a small portion, 1-2%, goes into the formation of wind (about 100 times the power that is stored in plants).[3] Wind develops when the sun's rays unevenly heat the air in the atmosphere. The majority of heating occurs at the equator, which receives the most direct rays. Those rays warm the equator air, which rises and moves north and south to the cooler regions. The air in the northern and southern hemispheres flows into the low-pressure area created at the equator by the rising hot air. At the same time, the earth is spinning creating a Coriolis force that shifts moving particles, such as the air, to the right in the northern hemisphere and left in the southern hemisphere. The uneven heating and the Coriolis forces together create the geostrophic winds, which are 1km above ground and are the overall prevailing winds in each region, as Figure 3 indicates.[4]

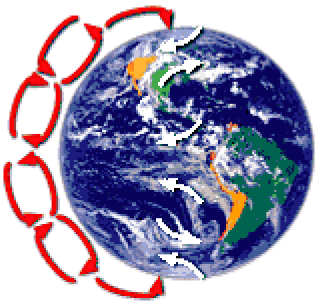

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Figure 3 **Prevailing wind directions arise from pressure differences (left arrows) and the Coriolis force (central arrows). (Danish Wind Industry Association, 2003)**

Geostrophic winds only give a very general idea of the direction of the wind at each latitude. At a given site, the elements of the landscape—hills, valleys, bodies of water, and other

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[3] Godfrey Boyle, *Renewable Energy: Power for a Sustainable Future*. (Oxford University Press, Oxford, 1996), p. 29.

obstacles—have a significant effect on winds as high up as 100m, although the upper atmosphere winds can pull along the lower winds and give them more power. The interaction between the wind and the roughness of the ground is known as wind shear. Shear is essentially friction on a large scale, but because air is not a rigid body, the air closer to the ground is affected more than that higher up. The velocity increases with height, as it becomes less affected by the roughness friction:

$$U \propto \ln(z/z_0) \qquad (1)$$

where U is velocity, z is the height, and z_0 is the roughness, essentially proportional to the overall height of the terrain (from 10^{-4}m over water to 1 m in cities).[5] Because the wind speed increases with height, wind turbines are mounted on high towers, although the height of an actual tower is limited by structural concerns and cost.

High speed winds can also be found in certain geographic locations. Bodies of water create significant winds. Sea-land breezes occur from uneven heating between the two areas. A body of water has a much higher heat capacity than land, which means it takes longer to either increase or decrease its temperature. During the day, the air over the land heats and rises, creating a low pressure for the sea air to flow into. This is analogous to the creation of geostrophic winds, where the land is the equator and the sea is the northern hemisphere. At night, the water stays warm while the land cools off quickly, and the air reverses its flow, now seaward.

Landscape and wind flow information for a given site are indispensable when deciding where to install a single wind turbine or a farm of turbines. The wind must have a minimum

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[4] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

[5] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

average speed (about 5-6 m/s) for any turbine to function, and what type, size, and style of wind turbine one chooses depends heavily on wind velocity data. When selecting a site, one gathers wind speed information on the number of hours per year that the wind at the site is at each speed. The data is essential for assessing the quality of the site (whether there is enough wind for a turbine to be cost effective) and the turbine type needed. A typical speed distribution curve is shown in Figure 4, which is usually a smooth distribution.[6]

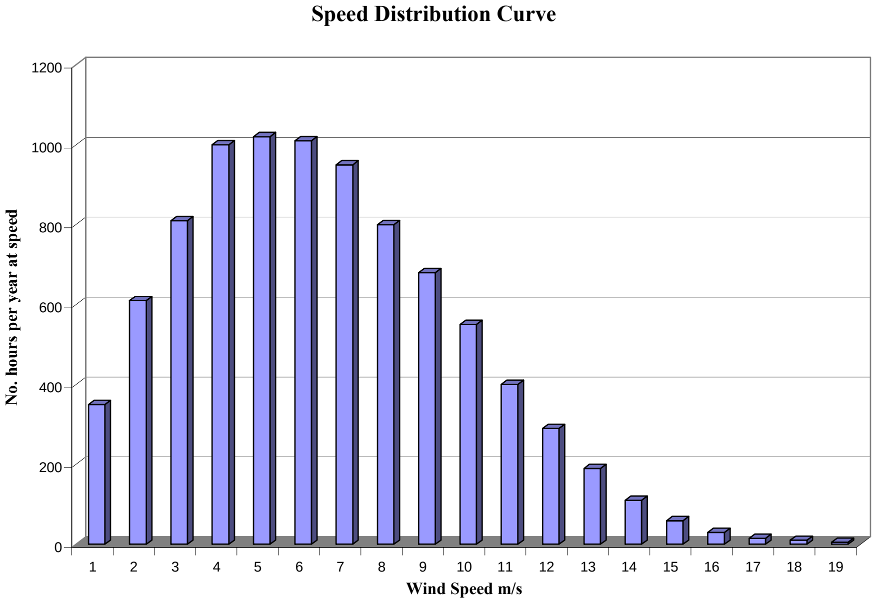

Figure 4 **A typical distribution of data representing the frequencies of wind speeds. Lower average wind speeds require a larger rotor to make up for the lower power.**

Power in the Wind

The importance of accurate wind speed data becomes clear when one understands how the speed affects the power. Consider a disk of area A with an air mass dm flowing through that area. In a time dt the mass will move a distance U dt, creating a cylinder of volume A U dt,

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[6] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).
Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

which has a mass d$m = A\ \rho\ U$ dt, where ρ is the density of air. The power contained in the moving mass is the time rate of change in kinetic energy, given by

$$P = d(KE)/dt = d(\tfrac{1}{2}mU^2)\big/dt = \tfrac{1}{2}U^2 dm/dt = \tfrac{1}{2}A\rho U^3 . \qquad \textbf{(2)}$$

Therefore, the power is proportional to the wind speed cubed.[7] It is important to know the wind speed precisely, because any error is magnified when calculating power. An important tool in analyzing wind turbine efficiencies and operation is a power duration curve, which includes the power in the wind on the vertical axis (accounting for the percentage of time spent at each wind speed) and the wind speed on the horizontal axis, such as in Figure 5.

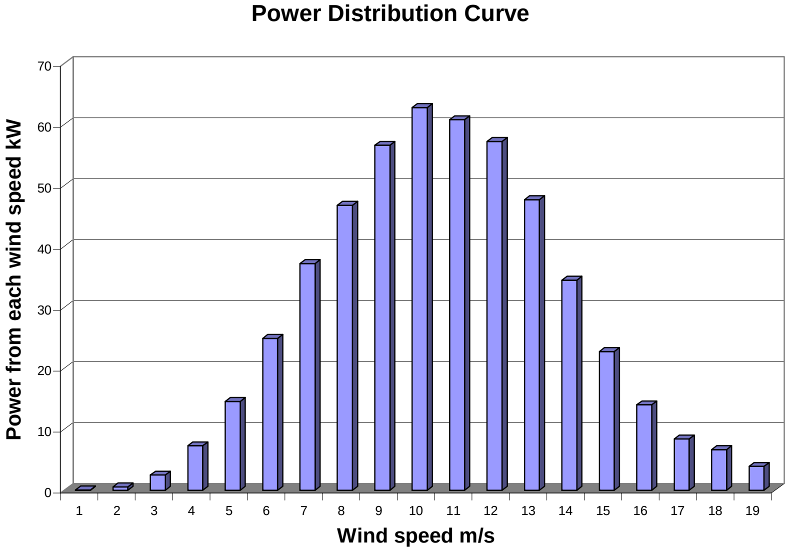

Figure 5 **Wind power distribution curve. Even though there might be fewer hours with 12 m/s winds than 7 m/s winds, the faster winds will still produce more power.**

A slightly more complicated analysis is needed to calculate the maximum power harnessed by a wind turbine because the blades necessarily interfere with the wind to extract its power. The following is highly idealized and makes several assumptions to simplify the algebra, basically that an infinite bladed rotor (with no losses from spaces between blades or from

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[7] Godfrey Boyle, *Renewable Energy: Power for a Sustainable Future*. (Oxford University Press, Oxford, 1996), p. 275.

imperfect airflow around the blades) encounters ideal and uniform airflow.[8] Nevertheless, the analysis provides information of a sufficient quality and introduces important concepts. It uses basic physics principles, simple fluid dynamics and a significant amount of algebra to arrive at an estimate of power output.

Consider once again the cylinder of air used to calculate the power in the wind. This is often referred to as a stream tube. The stream tube travels towards the wind turbine's rotor, as shown in Figure 6, with an initial velocity U and slows to U_1 (due to pressure changes) by the time it reaches the rotor. The rotor captures some of the energy so that air flowing out behind it moves even more slowly, at a velocity U_2, but the same amount (mass) of air coming towards the rotor also leaves behind the rotor. The wind turbine is not able to extract all of the wind's velocity; if it did, the air behind the turbine would stop, new air flowing in would not have anywhere to go, and excessive pressure would build up. The stream tube behind the turbine increases in volume (cross section) because the same mass is moving more slowly, forcing the air to expand to allow continued flow.

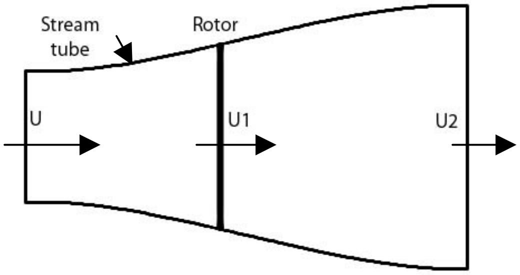

Figure 6 **Stream tube around a rotor, showing the velocities at various points. The initial velocity *U* is the free stream velocity, U_1 is the decreased velocity at the rotor, *and U2* is the slower velocity after the rotor.**

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[8] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

Because linear momentum is always conserved, a force must act on the wind to make it slow down. From Newton's third law, this force on the wind is equal to and opposite the thrust, T, the force of the wind on the turbine. The thrust force comes from a change in pressure as the wind passes the rotor and slows down. Conservation of linear momentum dictates that the thrust must be equal and opposite the change in momentum. We can look at the initial velocity of the air before the rotor, U, and the final velocity of the air after the rotor, U_2, along with the mass flow, to find the change in momentum:

$$T = (dm/dt)(U - U_2) = U(\rho AU) - U_2(\rho AU)_2 = \tfrac{1}{2}\,\rho\, A(U^2 - {U_2}^2)\,. \tag{3}$$

Further algebra (see Appendix A) shows that the power extracted from the air is

$$P = \tfrac{1}{2} A\rho U^3 4a(1-a)^2 \tag{4}$$

where a new value, the axial induction factor a, has been defined as

$$a \equiv (U - U_1)/U \tag{5}$$

the fractional decrease in the wind velocity once it has reached the rotor, due to a change in pressure (depending on how much energy the rotor captured to slow the wind). We can define a 'performance power coefficient,' C_p, as the ratio of the power in the rotor to the power in the wind:

$$C_p = 4a(1-a)^2\,. \tag{6}$$

The power coefficient indicates the efficiency of the turbine based solely on the stream tube concept, without accounting for non-ideal conditions and inevitable losses from the blades, the mechanics, and the electronics.

Taking the derivative of the power coefficient with respect to *a* and setting it equal to zero yields the axial induction factor of 1/3 which maximizes the efficiency. At this value of *a*, the power coefficient equals 16/27 ≈ 0.59. The restriction that no HAWT can extract more than 59% of the raw kinetic power in the wind is known at Betz's Limit. Figure 7 shows the efficiency relationship.[9]

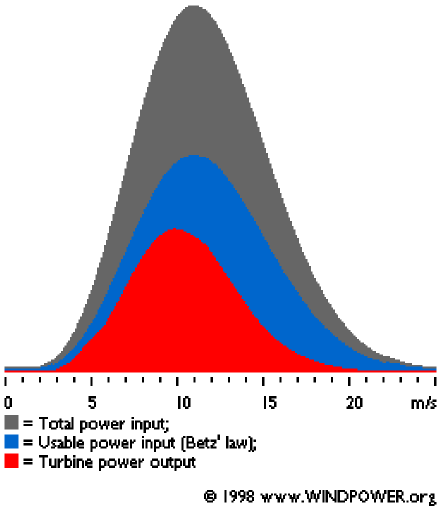

Figure 7 **The power in the wind (input), the usable power in the wind (Betz' law), and the actual output power of the turbine for a range of wind speeds. (Danish Wind Industry Association, 2003)**

An added complication that cannot be ignored even for simplified analysis, especially when calculating C_p, is the rotating wake created by the rotor. Based on the conservation of angular momentum, if the rotor gains angular momentum from the linear wind stream, then there must be some compensation, which is in the form of an opposite rotating wake (see Figure 8), so that the overall angular momentum does not change.

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[9] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

Figure 8 **A demonstration of the wake formed behind a wind turbine using smoke. (Danish Wind Industry Association, 2003)**

The blades have an angular velocity, *Ω*, relative to the linear wind, but an angular velocity relative to the wake of $\Omega + \omega$, where ω is the angular velocity of the wake relative to the linear wind. Another factor is introduced, the angular induction factor, defined as $a' \equiv \omega / 2\Omega$, which is a function of r. The pressure difference between the linear wind on the rotor and the rotating wake yields another equation for the thrust:

$$dT = 4a'(1+a')\rho r^3 \Omega^2 \pi\, dr \qquad (7)$$

which can be integrated over r to get the total thrust. (See Appendix B for derivation.) Once again looking only at the axial motion, one finds that, from Equations 3 and 5,

$$dT = \rho U^2 4a(1-a)\pi r dr\,. \qquad (8)$$

Although these equations for thrust were arrived at by analyzing linear and angular motion, they must still be equal to each other. Equating these equations gives

$$\frac{a(1-a)}{a'(1+a')} = \frac{\Omega^2 r^2}{U^2} = \lambda_r^{\ 2} \qquad (9)$$

where λ_r is the local speed ratio, i.e. the ratio of the rotor speed at radius r to the wind speed at r. When $r = R$ where R is the length of the blade, $\lambda_R = \Omega R/U = \lambda$, the tip speed ratio. The tip speed ratio is a very important parameter selected in the design of every wind turbine, because a greater tip speed ratio implies a greater power coefficient. The tip speed ratio can range from 5 to about 10 for electricity-generating applications.[10]

The power generated must now be recalculated to include the torque on the rotor from the wind making it turn. Using conservation of angular momentum, the torque, *Q*, for a small annular area must equal the change in angular momentum (see Appendix C for derivation):

$$dQ = 4a'(1-a)\tfrac{1}{2}\rho U\Omega\, r^2 2\pi r dr \qquad (10)$$

and the differential power becomes

$$P = \Omega dQ = \tfrac{1}{2}\rho A U^3 [\tfrac{8}{\lambda^2} a'(1-a)\lambda_r^{3} d\lambda_r] \qquad (11)$$

after using the definition of the local speed ratio. The power from any annular element depends on the axial and angular induction factors and on the tip speed ratio. The more complete power coefficient is

$$C_p = \frac{8}{\lambda^2}\int_0^{\lambda} a'(1-a)\lambda_r^{3} d\lambda_r . \qquad (12)$$

Figure 9 shows the power coefficient as a function of the tip speed ratio.[11]

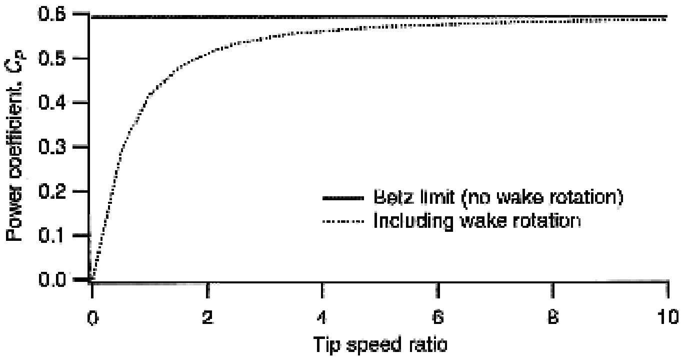

Figure 9 **Theoretical maximum power coefficient as a function of tip speed ratio, with and without wake rotation. This figure shows that higher λ are better for maximizing power. (Manwell et. al., 2002)**

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[10] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

[11] Ibid.

Aerodynamics of Blades

The above was an overall analysis looking at the whole turbine. Next, one must consider the ideal and possible blade shapes, given that a completely solid rotor is not possible. The current trend is to use three blades, a compromise between expense and ease of implementation. While one or two blades would be cheaper, they do not have the symmetry to naturally balance at all points in the rotation. The additional power gained with more than three blades is not worth the price of the extra blades.

A careful choice of the shape of the blades is crucial for maximum efficiency. Initially, wind turbines used blade shapes, known as airfoils, based on the wings of airplanes. Today's wind turbines still use airfoils, but they are now specially designed for use on rotors. Airfoils use the concept of lift, as opposed to drag, to harness the wind's motion. Blades that operate with lift, forces perpendicular to the direction of flow, are almost always more efficient than a drag machine (seen in Figure 10). Certain curved and rounded shapes have proven most efficient in employing lift. Below, in Figure 11, are some examples of the cross sections of blades used for wind turbines.

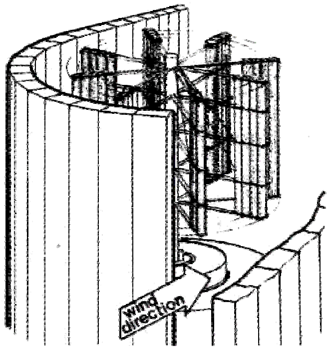

Figure 10 **Early Persian drag windmill, uses wind forces in the parallel direction—drag. (Manwell et. al, 2002)**

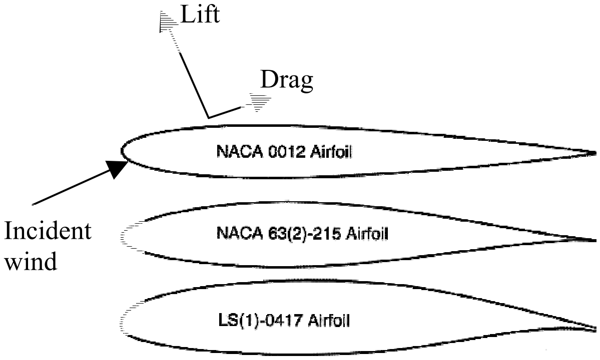

Figure 11 **Example airfoils (Manwell, et. al., 2002)**

The idea behind lift is that when the edge of the airfoil is angled very slightly out of the direction of the wind, the air moves more quickly on the downstream (upper) side creating a low pressure that essentially lifts the airfoil upward (see Figure 12). Bernoulli's law explains how faster flow implies lower pressure:

$$P + \tfrac{1}{2}\rho v^2 = const.. \qquad \textbf{(13)}$$

The first term is the static pressure and the second is the dynamic pressure. When the velocity, v, increases, there must be a corresponding decrease in the static pressure to maintain the constant, and vice versa. The equation can be thought of simply as conservation of energy ('pressure energy' lost to mechanical energy).[12]

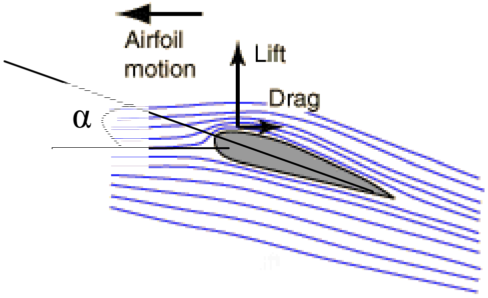

Figure 12 **Airflow around an airfoil. (http://hyperphysics.phy-astr.gsu.edu/hbase/pber.html)**

The amount of lift for a given airfoil depends heavily on the angle that it makes with the direction of the relative wind, known as the angle of attack, α. With a certain range, an increased angle of attack means increased lift, but also more drag, which detracts from the desired motion. When the angle of attack gets too large, turbulence develops and drag increases significantly, while lift is lost. The angle of attack on wind turbine blades can be changed either by creating a specific geometry for the blades along the span, or by allowing them to rotate around the axis

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[12] Martin O.L. Hansen, *Aerodynamics of Wind Turbines: Rotors, Loads and Structure*. (James & James Ltd., London, 2000) p. 14.

perpendicular to their cross sections (along the span). Changing the angle of attack is important to maintain a precise amount of lift so the rotor turns at a constant speed.

There are many parameters to characterize an airfoil as shown in Figure 13. The shape determines the performance of the airfoil, and often changes along the span of the blade. For instance, the blade is always much thicker at the base than at the tip, mostly for support.

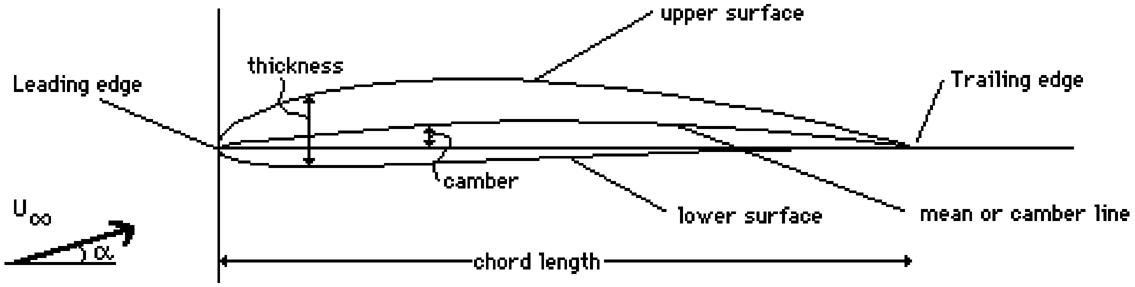

Figure 13 **The geometry and terms that define the shape of an airfoil, along with the wind U_∞ and angle of attack. (http://www.desktopaero.com/appliedaero/airfoils1/airfoilgeometry.html)**

The relative amount of lift and drag, indicated by the lift and drag coefficients, are a function of the shape and the angle of attack. Empirical or computational tests are used to determine the lift and drag for a given airfoil. The lift and drag coefficients (C_l and C_d) are defined as the lift or drag force per unit length divided by the dynamic force per unit length:

$$C_l = \frac{L/l}{\frac{1}{2}\rho U^2 c} \qquad (14)$$

$$Cd = \frac{D/l}{\frac{1}{2}\rho U^2 c} \qquad (15)$$

where c is the chord length, the distance from leading edge to trailing end. Figure 14 shows the lift and drag for a specific airfoil. Lift and drag characteristics can be used to describe the performance of a particular blade shape, or used to determine a blade shape for certain performance parameters. Two types of analysis, momentum theory (which has already been discussed) and blade element theory, which uses lift and drag forces, can be combined into blade element momentum theory (BEM) to gather either performance characteristics or optimum blade shapes.[13]

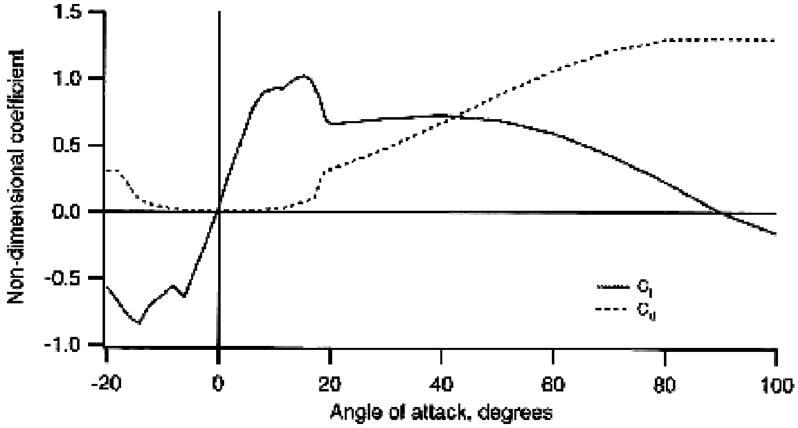

Figure 14 **Lift and Drag coefficients for S809 airfoil. (Manwell et. al., 2002)**

Blade element theory involves dividing the span of the blade into N sections or elements (see Figure 15) and determining the angles and forces along the span. The forces arise from the *relative* wind, to which lift is perpendicular and drag is parallel. The relative, or effective, wind is the vector sum of the wind velocity at the rotor, $U(1-a)$, and the wind velocity due to the rotation of the rotor, $\Omega r(1+a')$. Figure 16 shows this relationship.

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[13] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).
Martin O.L. Hansen, *Aerodynamics of Wind Turbines: Rotors, Loads and Structure*. (James & James (Science Publishers) Ltd., London, 2000).

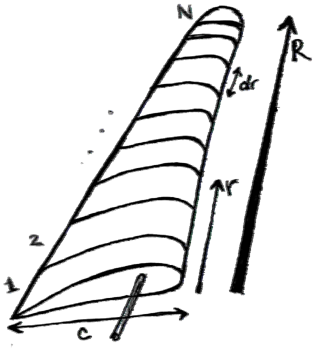

Figure 15 **Blade divided into N segments.**

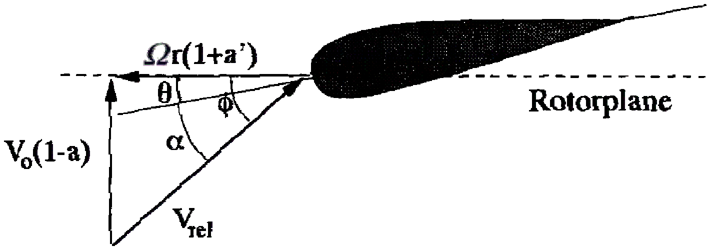

Figure 16 **Diagram of the vector addition of winds to find the relative wind, at an angle φ from the airfoil chord. (Hansen 2000)**

Angles and vectors are shown in Figure 17 to describe further the blade shape and motion information. It is important to note that the entire blade has a built-in twist along the span. An airplane has straight wings because it only moves linearly. The rotor on a wind turbine rotates, so that while the entire blade has the same angular velocity, the tangential velocity increases with increased radius, as does the relative wind. Similarly, the axial and angular induction factors are functions of radius *r* because they can change along the span of the blade. The blade must have a twist, and thus different angles of attack, so that the whole blade feels a consistent force (and limits stress on the blade).

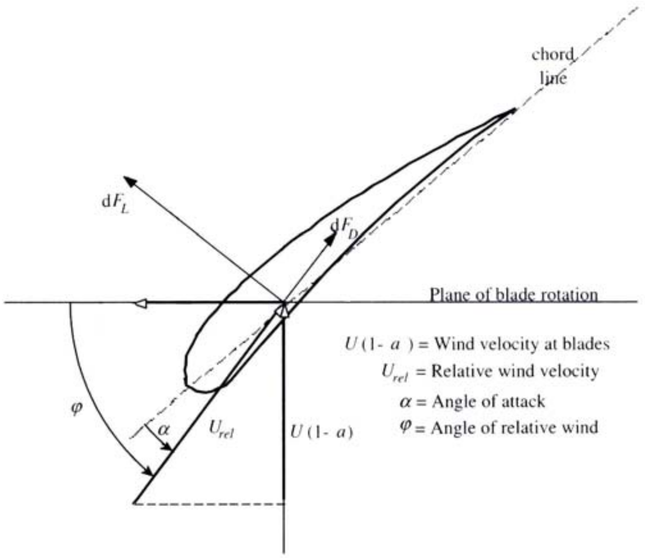

Figure 17 **Vectors and angles associated with a blade cross section. dF_L and dF_D are the lift and drag forces, respectively. (Manwell et. al, 2002, modified)**

Figure 17 contains information for several relationships that are used for choosing blade shape and optimizing power. Many of the equations needed for blade element analysis come directly from the geometry of figure. The thrust on an annular section *r* from the center is given by the vector addition of the lift and drag forces (adding the components perpendicular to the plane of rotation). The tangential force—which provides torque—is also found from the vector addition of lift and drag (adding the components parallel to the plane of rotation). Drag actually decreases torque, so one wants to minimize the C_d/C_l ratio.[14] (See Appendix D for actual equations.)

The blade element theory has yielded two equations, one for thrust and one for torque. Combining the two theories into BEM, one can find a blade shape for an ideal rotor. This task is usually accomplished with computer programs. Certain parameters are chosen, such as blades, *B*, length of blades, *R*, and the desired tip speed ratio, λ. Then an iteration process begins with an estimate of *a*, *a'*, and finds values for chord length and twist until a sufficient power coefficient

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[14] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

results from the blade shape. See Appendix E for optimum chord and twist calculation, and Appendix F for maximum power iteration. The optimum blade shape is usually not the one actually constructed. Variations along the blade are easier and less expensive to make if they are linear. Even though the ideal shapes do not vary linearly, a linear approximation still works well, especially since the ideal analysis is not perfect because of complications due to turbulence, high winds, and misalignment (rotor not facing the wind directly). [15]

Blade control must also be decided during early design stages, because the shape depends on the control method. A wind turbine will have either stationary, "stall-controlled," blades with a specially designed twist, or "pitch-controlled" blades that can rotate in their sockets to adjust for different wind conditions. The electricity requires that the rotor turn at a relatively constant speed, and because the wind is hardly ever constant, the blades must change the angle of attack and catch just the right amount of wind power to turn the rotor at a constant speed, which should be at the designed tip speed ratio λ.

The stall-controlled blades are able to regulate the rotational speed using a twist along the span so that when the wind speed starts getting too high, the lift drops at the base of the blade because the angle of attack increases. The drop in lift eventually becomes what is known as stall. At higher angles of attack, around 10-16 degrees, the wind no longer flows smoothly along the surface of the airfoil and becomes turbulent. The turbulence impairs lift because the air is no longer moving quickly enough to create a low pressure and, at the same time, drag increases. The drop in lift slowly crawls out to the tip of the blade as the wind speed continues to increase, using less of the high speed wind so that the rotor speed is constant. When full stall occurs, the blade stops (to avoid excessive load damage). The cut-out wind speeds (when the rotor shuts down) are in the range of

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[15] Martin O.L. Hansen, *Aerodynamics of Wind Turbines: Rotors, Loads and Structure*. (James & James (Science Publishers) Ltd., London, 2000).

20-32 m/s, while the cut-in speed (to start a turbine) is about 5 m/s. The advantage of this design is that few control mechanisms and electronics are need. However, the blade must be very carefully designed, and the necessary twist is not easy to manufacture.

Pitch-controlled turbines rotate their blades to smaller angles of attack, to get less lift, as wind speed increases. At the cut-out wind speed the blades turn their edges into the wind (essentially an angle of attack of zero) to eliminate lift and stop the rotor's motion. Unfortunately, the mechanical adjusting of the blade pitch occurs on a longer time scale than turbulent wind fluctuations, thus creating more power fluctuations. However, specially designed generators can correct for this problem. A third type of blade, the active stall, combines the two techniques. The blades rotate about the span-length axis to control rotor speed, but they use an increasing angle of attack as do stall-controlled blades. To stop the turbine, the blades are shifted into a high stall position, rather than being turned to a zero angle of attack.[16]

The theory behind blade design depends greatly on the concept that the wind is perpendicular to the plane of the rotor. Wind hardly ever maintains a single direction, although one direction may be more prevalent than others. A wind rose is a useful tool to visualize the prevailing wind directions, speed, power, and frequency at a given site. Figure 18 is an example of a wind rose for a viable site, because the wind is mostly in one direction. The turbine must be able to turn to face the wind direction, which is known as yawing. Precise electronic controls, motor, gears, and large ball bearings act together to make sure that the turbine remains facing the wind. A device measures the wind direction so the turbine knows where to turn. The controls and mechanics are not flawless and improperly yawed wind turbines can create excessive stress (gyroscopic loads) on the machine.

J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

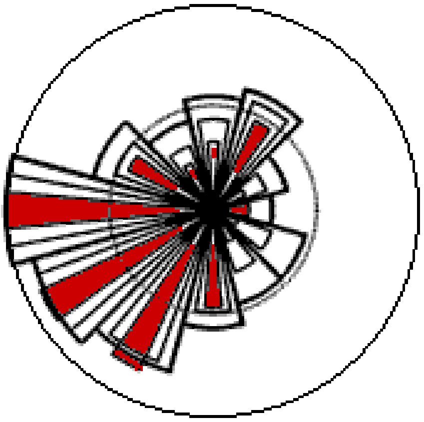

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Figure 18 **An example wind rose. The dark section is the power x frequency, the middle section is the wind speed x frequency, and the outer section is the wind frequency distribution. (Danish Wind Industry Association, 2003)**

Loads, Stress, and Fatigue

Aside from optimizing the blade shape and the yaw direction, a vital consideration in the construction of a wind turbine is the lifetime of the machine. Wind turbines are currently designed to last at least 20 years. The blades must be strong enough to withstand all the loads and stresses from gravity, wind, and dynamic interactions. Blades are carefully manufactured and then extensively tested to make sure they can achieve the desired lifespan. As opposed to a car engine and other mechanical devices, an efficient wind turbine runs about 90% of the time for twenty or more years. Certain loads will occur over 10^9 times during that period. The various loads on the blades and other parts of the turbine can be estimated and measured to understand the loads they must withstand.[17]

Types of loads are static, steady, cyclic, transient, impulsive, stochastic, and resonance induced. Static loads are constant and occur even with a non-moving turbine. These include steady wind and gravity. Steady loads are constant when the turbine is in motion and are caused by a

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[16] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

[17] Paul Rosenberg, *The Alternative Energy Handbook.* (The Fairmount Press, Librurn, GA, 1993).

steady wind. Cyclic loads are periodic, usually due to the rotation of the rotor. They occur from gravity, wind shear, yaw motion, and vibration of the structure. Transient loads are time varying with occasional oscillation. Braking by the inner gears and mechanics will cause this type of load. Impulsive loads are time varying on short scales, such as a blade being shadowed when passing the tower. Stochastic loads are random, usually around a constant mean value, and are primarily caused by turbulence. Resonance-induced loads, which are to be avoided as much as possible, occur when parts of the wind turbine are excited at their natural frequencies and then vibrate and can induce other parts to vibrate also, putting considerable stress on the turbine.[18]

Below are some examples of just a few types of loads. A blade is essentially a cantilevered beam and for bending moments can be expressed in the same way:

$$M(x) = \frac{w}{2}(L-x)^2 \tag{16}$$

where *M* is the bending moment—essentially an internal torque, *w* is the force per unit length, *L* is the length, and *x* is the distance from the root. Notice that most of the moment is at the root, hence the use of a thicker airfoil near the hub. A flapwise bending moment, which comes from the thrust forces is one of the more important considerations and can be expressed as follows:

$$M = \int_0^R r dT = \int_0^R r(4a(1-a)\rho \pi U^2 r dr)\,. \tag{17}$$

The thrust changes along the span and is usually stronger near the tip of the blade. The edgewise (lead-lag) bending moment comes from the tangential, lift forces and is characterized by a similar equation. A simple way of reducing the load from thrust is to cone the blades back towards the tower by an angle δ (as long as the blades are not in danger of hitting the tower). See Figure 19. When the blades are turning, they have a tendency to want to fly out of the plane of rotation. By

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[18] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley &

bending them slightly back in relation to the plane, that tendency to fly out of the plane of rotation that the blade feels is divided into two components, one along the blade and one perpendicular to the blade. The perpendicular component acts as an opposing force to balance out some of the thrust force.[19]

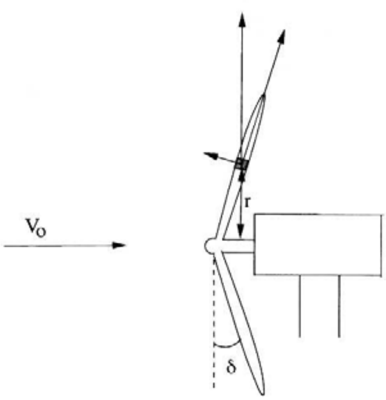

Figure 19 **Diagram of coning at an angle δ, direction in which the blade tends towards and its components are shown for a segment of the blade. (Hansen, 2000)**

Many other loads and their respective equations are needed to fully assess the stress a wind turbine might incur. In practice, extensive computer modeling and calculations of loads are made before any design is built. The reason that the loads on a turbine are extensively studied is shown below in Figure 20.

Sons Inc., West Sussex, 2002).

[19] Martin O.L. Hansen, *Aerodynamics of Wind Turbines: Rotors, Loads and Structure*. (James & James (Science Publishers) Ltd., London, 2000).

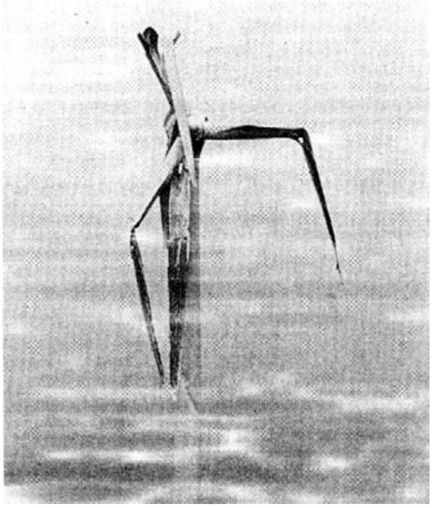

Figure 20 **A wind turbine that lost power to control it. The blades spun out of control and experienced excessive loads until the blades finally bent back far enough to hit the tower and break. (Hansen, 2000)**

To test how much stress a blade can withstand, the blades are subjected to sinusoidally varying loads until failure. Most frequently, the blade is attached securely at the base, as it would be on the turbine, and a machine is attached about halfway along the blade. The machine swings a weight up and down so that the blade oscillates. The test is performed for both edgewise and flapwise movement and continues until the blade breaks, often for longer than three months. Static tests are also performed, where more and more weight is slowly added until the blade breaks.

To achieve strong and stiff blades, a wooden frame is constructed, and then layers of composites such as fiberglass-reinforced plastic (GRP) are added. Glass fibers are woven into mats and polyesters or epoxies are added to strengthen and give support. Unlike metals, composites have high strength and stiffness to weight ratios and are also electrical insulators, a useful property in the presence of lightening. Only the outermost part of the blade needs to fit the chosen airfoil shape, but the smoothness of the surface is critical for predicted performance. Dust and dirt that sometimes collect on a blade over time can eventually reduce the efficiency by 40%.[20]

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[20] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

Gearbox

As the rotor does its job by turning at the designated wind speed, the rotor shaft connected to it turns inside the nacelle. The nacelle is a large box at the top of the tower that holds all the mechanics and electronics that are needed to generate electricity. The main components are the gearbox, the generator, the cooling system, the yawing system, and the brakes, as can be seen in Figure 21.

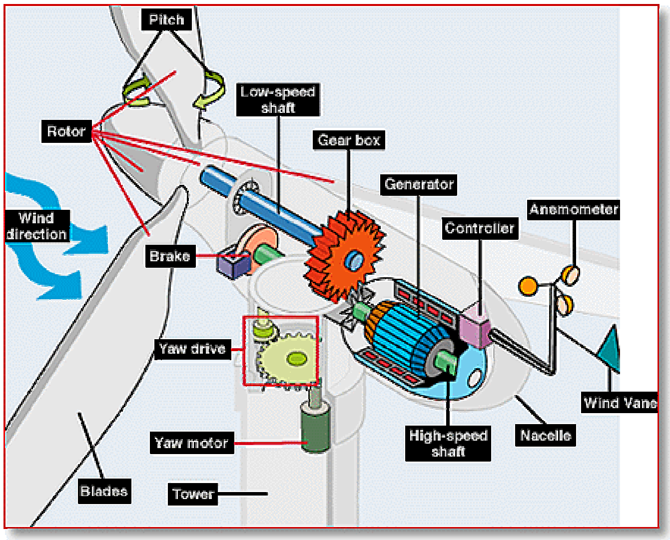

Figure 21 **Diagram of the components inside the nacelle of a wind turbine.**

(http://www.powerhousetv.com/stellent2/groups/public/documents/pub/phtv_eb_re_000315.hcsp)

The first step is to convert the rotor shaft speed into a speed that can be used by the generator. Most generators operate at significantly higher speeds than those at which a wind turbine rotor can reasonably operate. A rotor would have a tip speed greater than the speed of sound if it were to accommodate a standard generator.[21] Gears are particularly useful for changing shaft speed. The gear connected to the rotor shaft has more teeth than the gear connected to the

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[21] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

generator. In transferring from a larger gear to a smaller gear, the rotational speed increases and the torque drops, while total power remains the same:

$$Q_1\Omega_1 = Q_2\Omega_2 = P. \quad (18)$$

The ratio of the shaft speed is equal to the inverse ratio of the number of teeth on the connecting gears:[22]

$$\Omega_1 / \Omega_2 = N_2 / N_1. \quad (19)$$

To understand the ratios, consider that in one full rotation of the larger gear, the smaller gear rotates many more times, and thus the shaft must move much faster. The gear ratio of Carleton College's 1.65 MW wind turbine is 1:84.3, so that when the rotor is operating at its rated speed of 14.4 rpm, the generator shaft is turning at about 1214 rpm. Some newer turbines use a direct connection from the rotor to a special generator. Losses and expenses from the gears are removed and tests so far have shown comparable efficiency to the traditional gear system.[23]

Generators

A generator uses the rotor shaft speed to convert mechanical energy to electrical energy with the use of magnets. As I mentioned earlier, wind turbines are like inverse fans, so the generator is essentially a motor that is running backwards. In fact, with the right input generators can often be made to run like a motor. The main concern with generators in wind turbines is that they must produce electricity compatible with that in the electrical grid for the given site. In America, the grid is at 60 Hz, while in most parts of Europe the grid runs at 50Hz. The electricity must have the same

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[22] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

[23] Anders Grauers, IEEE Transactions on Energy Convention **11** (3), 650 (1996).

characteristics, be of sufficient quality, and be connected in such a way as to not interrupt the existing current flow.

Wind turbines have one of two types of generators: synchronous or the more common asynchronous (induction). Usually, current flowing through solenoids coiled around iron cores provides the magnetic field. Only a few generators use permanent magnets, since the required size makes them prohibitively expensive, and over time they demagnetize. However, some studies do show viability for this type of magnet.[24] Alternating current generators use Faraday's law of induction to produce the electricity. The basic concept is that when a loop of wire is in a magnetic field—generated by the coils—there is a magnetic flux through that loop according to

$$\Phi_{magnet} = \int \vec{B} \cdot d\vec{A} = BA\cos\theta \tag{20}$$

where θ is the angle between the area vector A and the magnetic field B. If the loop is rotated, such as by the rotor shaft, the angle changes according to the angular velocity of the motion, ω, and thus so does the magnetic flux. The induced, oscillating, electromagnetic force equals the negative of the change in flux:

$$emf = -\frac{d\Phi}{dt} = BA\omega\sin(\omega t)\,. \tag{21}$$

The loop now has current running through it, which can be sent to the grid for use.

Naturally, actual generators are more complicated. For instance, to connect the generated electricity to the grid, it must have currents in phase with the grid, otherwise a power surge and damage occur. A three-phase generator outputs three power lines with the alternating current running in parallel, while the phase for each is shifted 1/3 of the cycle more than the previous line. Figure 22 shows how the sum of the voltages at any given time is zero. A 2-pole generator has three magnets, each connected to the current phases. The coil magnets are in either a Y shape, or a

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[24] A.C. Williamson E. Spooner, IEE Proc.-Electr. Appl. **143** (1), 1 (1996).

delta shape,[25] as shown below in Figure 23. As a pole or electromagnet in the center of the coils rotates, the three magnets alternate between north and south, so that as the current fluctuates there is one 'full' north and one 'full' south among the three coils, hence the name "2-pole."[26] While this configuration describes a synchronous generator, some modifications need to be made for the typical asynchronous generators used in wind turbines. Asynchronous generators were originally designed as reliable and inexpensive motors, but as generators, they have additional properties beneficial for wind turbines.

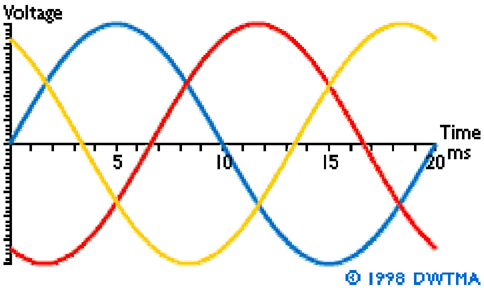

Figure 22 **Voltage vs. time graph of a three phase system. The net voltage equals zero at all times. (Danish Wind Industry Association, 2003)**

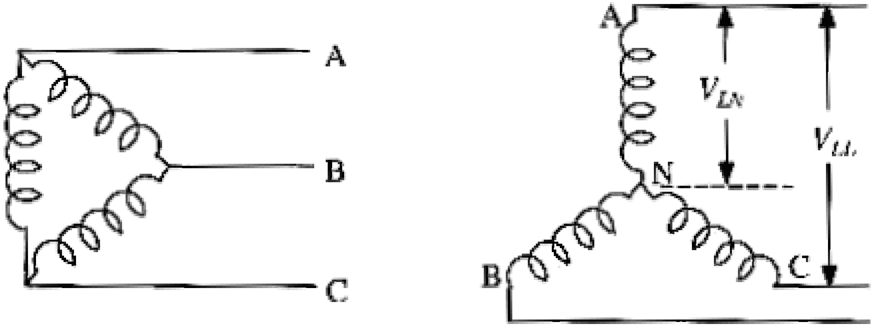

Figure 23 **Delta coils—connected between the phase conductors—on the left, Y coils on the right—each connected to a phase conductor, and all connected to neutral. The lettered lines are the three phase conductors (and N=neutral for the Y). V_{LN} is line-to-neutral, V_{LL} is line-to-line voltage . (Manwell et. al., 2002)**

Generators in wind turbines are usually 4-pole, which means they have six magnets in a circle, and thus two full north and two full south poles, at any given time. See Figure 24 for a

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[25] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

stylized version. A larger number of poles means the generator's rotor does need to move as far to get to the next pole, so it can turn at a slower speed, thus reducing the size of the gearing (although it also increases the expense of the generator). The stationary part of the mechanism, in this case the coils, is known as the stator. The rotating part is the rotor which produces the magnetic field to induce a current. As the rotor turns, it induces alternating currents in the coils, thus acting as a generator of electricity. The rotational speed of the rotor, when acting as a motor or a synchronous generator, equals the speed of the rotating magnetic field, which is found from the equation $\frac{60\upsilon}{p/2}$, where υ is the frequency of the grid, p is the number of poles and 60 converts from r.p.s. to r.p.m. There is a factor of two because a 2-pole system must go through a full cycle (rotation), whereas a 4-pole system rotates only ½ the cycle before it sees the same pole. The synchronous speed is usually 1500 rpm (50Hz) or 1800 rpm (60Hz). The stator initially receives current from the grid and creates a magnetic field. That field induces a current in the bars of the rotor. The rotor current generates its own magnetic field that, in turn, induces a current in the stator that is sent to the grid.

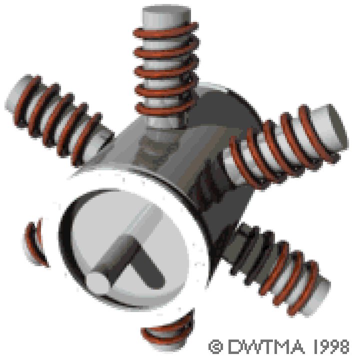

Figure 24 **Stylized version of a 4-pole generator. (Danish Wind Industry Association, 2003)**

The above figures and explanation are an idealization of an induction generator. The type often used in wind turbines have a "cage" rotor, with bars of copper or aluminum connected electrically with end rings, as shown in Figure 25. The rotor is placed in the middle of the 4-pole

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[26] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site:

stator and allowed to rotate. However, it does not run at the synchronous speed, i.e. the speed of the magnetic field. When the magnetic field and the rotor are turning at the same speed, no induction occurs. However, if the rotor turns at a slightly greater speed, then induction can occur, and current will flow in the stator. The rotational speed at which maximum power occurs is only about 1-2% higher than the synchronous speed. The percent difference is called the slip. A generator that will idle at 1500 rpm will produce the most power when the rotor is at 1515 rpm.[27]

Figure 25 **The cage part of 'cage wound' generator. (Danish Wind Industry Association, 2003)**

The stator actually consists of many layers of steel sheets with dozens of individual magnets, as shown in Figure 26. The number of poles can be changed simply by grouping the magnets into a different number of bunches. The reason for the 'many' stator magnets is that they minimize the air gap between the stator and the rotor, given that magnetic flux through air is not very strong. This design of variable poles, also allows greater efficiency at lower wind speeds. That some generators can change the number of poles means the generator's rotor can run at a lower speed and thus so can the blades. [28] At lower wind speeds, a lower rotor speed is more aerodynamically efficient, assuming that the generator can adjust for it. Calculations reveal that variable speed generators can be just as efficient in wind turbine applications as regular generators.[29]

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http://www.windpower.org/en/tour/wres/index.htm

[27] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

[28] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

[29] Anders Grauers, IEEE Transactions on Energy Convention **11** (3), 650 (1996).

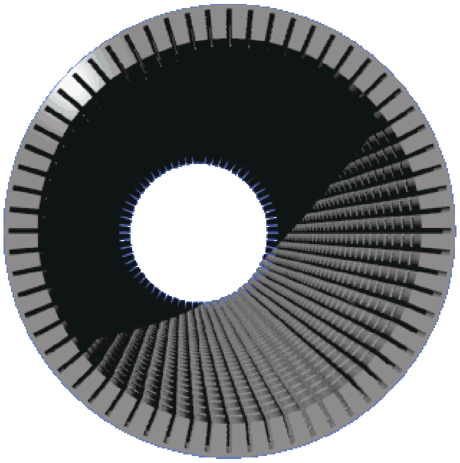

Figure 26 **Stator made of steel sheets and divided into many individual magnets (w/o windings). (Danish Wind Industry Association, 2003)**

Asynchronous generators are useful because the slip allows them to decrease or increase speed if the torque varies, which means less wear on the gearbox and higher quality output. The slip depends directly on the internal resistance of the rotor, and so increasing the resistance means an increase in slip. A technique known as Opti Slip® developed by Vestas, one of the leading manufactures of wind turbines, provides a method of changing the slip on a generator. Resistors are mounted on the rotor, as is an electronic control system. To communicate the desired amount of slip (i.e. resistance) according to the rotor speed, a stationary optical fiber sends a signal to the electronics as they pass by, and they change the resistance.[30] The variability of the slip helps compensate for the relatively slow reaction time of a pitch and yaw controlled system to quick variations in wind speed and direction. In this way, the blade rotor does not have to run continually at a precise speed to ensure that the generator outputs high quality electricity.

Electronics and the Electrical Connection

For most wind turbines, connection to the grid is direct from the generator, after using a transformer to step up the voltage to match that of the grid. Variable speed wind turbines require an indirect connection because the current generated is of varying frequency. Converting to DC and then restoring back to a fixed frequency AC corrects for this problem. Variable speed turbines can capture slightly more of the wind's power than fixed speed turbines and the indirect connection is often of higher quality. However, the additional power gained is offset by extra cost of the power electronics and losses during the conversions.[31]

A device checks periodically—about 7680 times per second—the power quality of the produced current. It calculates the stability and uses capacitors to adjust the reactive power—the phase angle between the voltage and the current. The current produced must match that of the grid and have a consistent sinusoidal shape. The current must also be connected to the grid at the right time. Connecting it with an ordinary switch would cause a surge of current into the turbine to start it and then a surge out of the turbine once the generator started up. Instead, turbines are connected 'softly' with the help of thyristors. A thyristor is a switch that can be controlled electrically with a small amount of current and allow the large voltage connection or disconnection to be gradual. Thyristors are a four layer (pnpn) semi-conducting transistor type devices—they are the electronics that control light dimmers. A mechanical switch takes over once the connection has been made to reduce the 1-2% heat losses that are inevitable with thryistors. [32]

Issues with Wind Energy

Wind is far from a prefect energy source. Although the 'fuel' is clean and renewable, it is neither reliable nor easy to harness. Siting is essential to an efficient turbine. The location must offer sufficient yet not overpowering wind speeds at most times. The type of wind turbine—height,

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[30] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

[31] Anders Grauers, *Efficiency of three wind energy generator systems*, IEEE Transactions on Energy Convention **11** (3), 650 (1996).

[32] Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

blade number, generator type, rated speed, total power—must be chosen to match its environment. The siting requirements are very particular and have several negatives. For instance, because wind turbines cannot be put just anywhere, sometimes the best places to put them interfere with the visual sites or the wildlife.

Electromagnetic interference—such as with T.V. reception—was once a problem because the earliest blades were made of metal, but now that blades are made of composites, that has become less of an issue. Lightning still poses problems, even with the use of composites for the blades. Noise from the blades is often a complaint, although newer designs minimize noise, and locations are usually not near any human habitation. Measurements of the noise produced are difficult to make because the background level is almost at that of the turbine itself. Currently, designers continue to explore improvements in making wind turbines more efficient.

The Future

The latest developments involve improved generators (such as OptiSlip® and variable speed mentioned above), and off-shore wind farms. The off-shore wind farms are particularly attractive because water has a very low level of roughness (it interferes less with the wind) and wind speeds in general are higher and less turbulent off shore. Wind turbines off-shore can therefore be much larger, on the order of 4 MW, compared to less than 2 MW. The problems of assembly, especially establishing a foundation and the connection to a grid, are the main obstacles, although many of these have been addressed and economically viable solutions are already in use. Wind turbines enable the use of a clean and renewable energy resource. Physicists and engineers are clearly facing and overcoming the challenge of dealing with what is also an unconventional and unpredictable energy source.

Acknowledgements

I would like to thank my advisor Steve Parker for his excellent suggestions, as well as Kris Wedding and Matt Mewes for also helping me improve this paper. I would also like to acknowledge all the physics faculty at Carleton College, and my fellow majors, for giving me a valuable background in physics.

Appendix

A. The power in the rotor using the stream tube:

The thrust is the change in velocity multiplied by the mass flow:

$$T = (dm/dt)(U - U_2) = \tfrac{1}{2}\rho A(U^2 - U_2{}^2)\,. \qquad (1)$$

Using known fluid flow techniques and the Bernoulli function, one can show that the flow velocity at the rotor is the average of the velocities before and after:

$$U_1 = (U + U_2)/2\,. \qquad (2)$$

Although algebra involving pressures and densities can prove this relationship, it should seem reasonable as only an assumption.

In order to characterize wind turbines, a variable known as the axial induction factor, *a*, is often introduced, where

$$a = (U - U_1)/U \qquad (3)$$

is the fractional decrease in the wind velocity when it reaches the rotor. The power transferred to the wind turbine is given by

$$P = TU == \tfrac{1}{2}A\rho(U^2 - U_2{}^2)U_1\,. \qquad (4)$$

A little algebra using Equation 2 and 3 shows that

$$U_1 = U(1 - a) \qquad (5)$$

and that

$$U_2 = U(1-2a)\,. \tag{6}$$

Putting these values into the power equations gives the power in the rotor[33]:

$$P = \tfrac{1}{2} A\rho U^3 4a(1-a)^2\,. \tag{7}$$

B. Thrust with wake rotation:

Since $\Delta T = \Delta p\, A$, we can calculate the change in pressure with rotational motion. A common fluid dynamics equation (Bernoulli's equation) says the change in pressure equals the dynamic pressure:

$$\Delta p = \tfrac{1}{2}\rho v^2\,. \tag{1}$$

The rotational velocity ωr can be inserted for v, but since there are two different angular velocities, ω and Ω, we can use a combination (the complete derivation can be found elsewhere):

$$dT = \rho r^2 \omega(\Omega + \tfrac{1}{2}\omega)2\pi r dr\,. \tag{2}$$

After using the definitions for the axial induction factors, the thrust can now be expressed using the rotational motion:

$$dT = 4a'(1+a')\,\tfrac{1}{2}\rho r^2 \Omega^2\, 2\pi r dr\,. \tag{3}$$

C. Finding the torque

Once again using conservation of angular momentum, the torque, Q, for a small annular area must equal the change in angular momentum, $dL/dt = d(r \times p)/dt$:

$$Q = r \times dp/dt = r\dot{m}v = \dot{m}(\omega r)r \tag{1}$$

where ω is the change in the angular velocity (contributing to the change in angular momentum).

Putting in the values for the change in mass, the differential torque is thus

$$dQ = d\dot{m}(\omega r)r = \rho U 2\pi r dr(\omega r)r\,. \qquad (2)$$

Substituting *a* and *a'* the torque becomes

$$dQ = 4a'(1-a)\tfrac{1}{2}\rho U\Omega\, r^2 2\pi r dr\,. \qquad (3)$$

D. Equations for blade element theory:

The relationship between the two induction factors and the angle of the relative wind is

$$\tan\varphi = \frac{(1-a)U}{(1+a')\Omega r}\,. \qquad (1)$$

The normal force (thrust) on a section *r* from the center is given by

$$dF_N = B\tfrac{1}{2}\rho U_{rel}^{\,2}(C_l\cos\varphi + C_d\sin\varphi)c\,dr \qquad (2)$$

for a rotor with *B* blades, where the cosine and sine terms are the components of the lift and drag forces, and *c* is the chord length. The torque is

$$dQ = B\,r\,dF_T = B\,\tfrac{1}{2}\,\rho U_{rel}^{\,2}(C_l\sin\varphi - C_d\cos\varphi)\,c\,r dr\,. \qquad (3)$$

E. Equations for ideal blade chord and twist:

We assume that there is no wake rotation, $a' = 0$, and that drag is negligent, $C_d = 0$. We will also assume that $a = 1/3$, which allows a maximization of the power coefficient. From momentum theory, the thrust is thus

$$dT = \rho U^2\,\tfrac{8}{9}\pi r dr \qquad (1)$$

and from blade element theory the thrust is

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[33] J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

$$dF_N = B \tfrac{1}{2} \rho U_{rel}^{\;2} (C_l \cos\varphi) c\, dr\,. \tag{2}$$

Equating these two values for thrust, and expressing U_{rel} in terms of U (free stream velocity):

$$U_{rel} = U(1-a)/\sin\varphi = \frac{2U}{3\sin\varphi} \tag{3}$$

one finds the relation:

$$\frac{C_l B c}{4\pi r} = \tan\varphi \sin\varphi\,. \tag{4}$$

Figure 16 offers another equation for tan φ, still assuming that a = 1/3 and a' = 0:

$$\tan\varphi = \frac{2}{3\lambda_r}\,. \tag{5}$$

This can be inserted into Equation 38, to yield a result for the chord of the blade:

$$c = \frac{8\pi r \sin\varphi}{3BC_l\lambda_r}\;. \tag{6}$$

The twist angle derives from the section pitch angle θ_p, and the section pitch angle is the difference between the angle of the relative wind, φ, and the angle of attack: $\theta_p = \alpha - \varphi$.

F. Optimizing blade shape and performance:

One begins by finding expressions for the axial and angular induction factors by equating the equations for torque and the equations for thrust:

$$\frac{a'}{(1-a)} = \frac{\sigma' C_l}{4\lambda_r \sin\varphi} \tag{1}$$

$$\frac{a}{(1-a)} = \frac{\sigma' C_l \cos\varphi}{4\sin^2\varphi}\,. \tag{2}$$

A new symbol has been introduced, $\sigma' = Bc / (2\pi r)$, the local solidity. More equations will explain how we determine the flow conditions for each of the N sections. An iterative algorithm is used to find the forces and power:

1. Guess a and a'
2. Find φ using $\tan\varphi = \dfrac{(1-a)U}{(1+a')\Omega r}$ (Equation 1, Appendix D)
3. Find angle of attack: $\alpha = \theta - \varphi$
4. Find C_l and C_d from empirical data
5. Calculate C_n and C_t, the coefficients for the net thrust and normal (torque) forces
6. Recalculate a and a', using equations

$$a = \frac{1}{[(4\sin^2\varphi/\sigma C_n)+1]} \tag{3}$$

and

$$a' = \frac{1}{[(4\sin\varphi\cos\varphi/\sigma C_t)-1]} \tag{4}$$

7. If a and a' have changed too much, go to step 2
8. Compute forces on each blade segment and the power coefficient,

$$C_p = \frac{8}{\lambda^2}\int_{\lambda_h}^{\lambda} \lambda_r{}^3 a'(1-a)[1-(C_d/C_l)\cot\varphi]d\lambda_r \tag{5}$$

(The new power coefficient has been derived from various equations above.) It is easy to see how a computer is a useful tool for this process.

Annotated Bibliography

Paula Berinstein, *Alternative Energy: Fact, Statistics, and Issues*. (Oryx Press, Westport, CT, 2001).

An excellent resource for history, statistics and general facts about a range of alternative energies. Includes mostly cost analysis, but not much in the realm of how wind energy systems actually work.

Godfrey Boyle, *Renewable Energy: Power for a Sustainable Future*. (Oxford University Press, Oxford, 1996).

Covers a wide range of renewable/alternative energies. The wind energy chapter includes a case study of a wind farm, and basics about wind. Includes very concise figures and assesses a variety of turbine types. Basics of HAWT operation are also covered along with force diagrams and occasional equations. Environmental and economic concerns are also addressed.

Jennifer Carless, *Renewable Energy*. (Walker and Company, New York, 1993).

This book was of limited usefulness to me. It has a short section on wind energy, which dealt primarily with the current status of wind energy around the world and the economics of wind turbines.

Danish Wind Industry Association, *Guided Tour on Wind Energy.* (1997-2003) Internet Site: http://www.windpower.org/en/tour/wres/index.htm

An excellent resource for all the basics of how wind turbines work. It includes simple explanations of almost every aspect of wind energy. Has clear pictures, interactive options with calculators of various concepts such as wind speed curves (user can input values), and videos. I used this resource extensively to understand the basics, although it does not take a physics/math approach to the analysis. I also included many of the pictures from the website.

Anders Grauers, *Efficiency of three wind energy generator systems*, IEEE Transactions on Energy Convention **11** (3), 650 (1996).

A study comparing different types of generator systems for efficiency (rotor shaft to grid). Discusses a conventional 4-pole induction generator, a variable speed synchronous generator, and a directly driven (no gears) variable speed generator. Concludes that variable speed generators are almost as efficient (on average) as fixed speed, and directly driven generators can be more efficient than the conventional gear system.

Martin O.L. Hansen, *Aerodynamics of Wind Turbines: Rotors, Loads and Structure*. (James & James (Science Publishers) Ltd., London, 2000).

A great source of aerodynamic information for wind turbines. Discusses the aerodynamics of the rotor, the blades, and the loads in physics/mathematical terms. I used this for much of the aerodynamic and load sections.

J.G. McGowan J.F. Manwell, A.L. Rogers, *Wind Energy Explained: Theory, Design and Application*. (John Wiley & Sons Inc., West Sussex, 2002).

This is one of best resources I found. It is a comprehensive look at all aspects of wind turbines (focusing on HAWT designs), with a physics/engineering emphasis. Many of the equations and derivations I used came from this book. It is very clear, although organized somewhat haphazardly. Gives basic background for the various systems—such as electricity and magnetism for the generator—as well as how those systems apply to the actual wind turbine.

Martin T. Katzman, *Solar and Wind Energy*. (Rowman & Allanheld, Totowa, NJ, 1984).

Discusses several types of alternative energies. Focuses mostly on the economics and the effects of 'farming the wind.' Not very useful in terms of how wind energy works.

Paul Rosenberg, *The Alternative Energy Handbook*. (The Fairmount Press, Librurn, GA, 1993).

This book is a useful short look at the history of wind power, different types of wind turbines, and the basics of operation.

James L. Schefter, *Capturing Energy from the Wind*. (NASA Scientific and Technical Info Branch, Washington, D.C., 1982).

A short book on small scale wind turbines and how they operate. Also includes some history of wind energy and a 'build you own' approach to turbines.

Shikha, T.S. Bhatti, D.P. Kothari, *Aspects of Technological Development of Wind Turbines*, J. Energy Engineering, 81 (2003).

A review of wind turbine technology, improvements that have been made, and others that should be made to increase the cost effectiveness of wind turbines. It is essentially a seven page overview of wind turbines with a focus on possible problems.

J.G. Slootweg, *Dynamic Modeling of a Wind Turbine with Doubly Fed Induction Generator,* IEEE, 644 (2001).

A very specific paper that looks at a different type of generator than the one I explained in detail. It focuses on the influence wind turbines may one day have on the electrical grid system. It also includes a complicated mathematical model of a hypothetical turbine.

Bent Sorensen, *Renewable Energy: Its physics, engineering, use, environmental impacts, economy and planning aspects*, Second ed. (Academic Press, San Diego, CA, 2000).

A large book covering a wide range of alternative energies. The book was not easy to navigate because information such as wind, aerodynamics, and electronics were interspersed with information that does not apply to wind energy systems (e.g. photovoltaics). The information itself, though, is very detailed and physics-oriented. I limited my use of this book because of the unsuitable organization and because other books had the same information in a more comprehensible style.

Poul Sorensen, Jan Skaarup, Florian Iov, *Dynamic Phase Compensation of Wind Turbines*, Nordic Wind Power Conference (2004).

A short paper dealing with grid connections. It was too specific to be of much use, although I did gather some information on the thyristors and capacitors used to smooth out the electricity flow.

David A. Spera, in *Wind Turbine Technology: Fundamental Concepts of Wind Turbine Engineering* (ASME Press, New York, 1994).

This was an interesting resource from an engineering point of view. I did not use it because more recent books provided updated and more comprehensible information. This book has a "build-it-yourself" approach to personal wind turbines.

A.C. Williamson E. Spooner, *Direct coupled, permanent magnet generators for wind turbine applications*, IEE Proc.-Electr. Appl. **143** (1), 1 (1996).

A look at how directly coupling the rotor shaft to a permanent magnet generator is possible with the use of a special type of generator. The details and explanation of the generator are good, but too specific for use in this paper, although it did help me understand how traditional generators work.

D.T. Swift-Hook, in *Wind Energy and the Environment* (Peter Peregrinus Ltd., London, 1989).

This book is a collection of many short papers concerning a variety of aspects of wind turbines' interaction with the environment—such as noise emission, blade detachment, EM interference, etc. The information is all too specific for use in this paper.